

Hamiltonian field theoretical model for a light quark condensate

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Abstract

I propose an alternative Hamiltonian field theoretical model for a light quark condensate that is compatible with QCD in the deep infrared. Key electroweak data on flavourless pseudoscalar mesons are used for necessary renormalizations. Light quark inertial masses are redefined in a new and broader theoretical context.

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I. INTRODUCTION

The aim of this paper is to reconsider again the old pre-QCD Nambu-Jona-Lasinio(NJL) ideas, but this time in the light of the new insights gained in the last 10 years. We shall make appropriate use of contemporary methods and ideas on boson and fermion condensates that proved to be so useful in both nuclear physics and condensed matter physics. The phenomena associated with broken chiral-flavour symmetries and chiral anomalies are indeed to be ultimately related to the physical nature and structure of a presumed quark-gluon quantum vacuum.

We assume the existence of a u,d fermion condensate and work from there, using mathematical methods more or less common to all fermionic systems at zero temperature. Our basic degrees of freedom are chiral QCDu,d,s-quarks but only in colour singlet combinations.

Their couplings to the leptons and gammas are assumed to be primarily as prescribed by the Standard Model (SM).

Updating the NJL idea, we design an effective Hamiltonian to be worked out under the assumption that it has a stable Dirac-Hartree-Fock-Bogoliubov (DHFB) state as its (approximate) ground state. Necessary renormalizations are carried out using experimental meson masses and electroweak data. Applications and detailed numerical results will be published elsewhere.

II. THE RELEVANT EFFECTIVE HAMILTONIAN

We begin by making the standard symmetry assumptions[1,2,3]. The internal symmetry that phenomenology suggests as the most relevant to the analysis of the lightest quark sector is the chiral-flavour symmetry of the form

$$G : SU_{NL} \otimes SU_{NR} \otimes U_{1L} \otimes U_{1R} \quad (1)$$

with $N=2,3$. The chiral-flavour symmetry G undergoes various quantum mechanical symmetry breakings [2,3] that reduces it down to flavour groups

$$G \implies H : SU_{NL+R} \otimes U_{1L+R} \quad (2)$$

Thus G is a symmetry of the Hamiltonian (or equations of motion) whereas H is a symmetry of the quantum vacuum. There are two well-understood kinds of symmetry breakings that achieve this, both traceable to the quantum vacuum[2,3,4]. The first kind is due to the existence of a specific classically definable Landau-like vacuum long range order (LRO)[5], to be defined more precisely in the following sections. It breaks the chiral SU_N (SU_N) part of the algebra (1), leading to the emergence of the right set of Goldstones(viz. 3 and 8). The other kind of quantum mechanical symmetry breaking removes the axial U_{1L-R} symmetry of the Hamiltonian, as a consequence of the axial anomaly. The latter can be interpreted as simply a consequence of the physical existence of the Dirac vacuum[4]. This

implies that there are no chiral doublets in the hadronic phase of QCD in which we live. This information is the input used in designing the effective Hamiltonian of this paper.

We choose our basic degrees of freedom to be included in this effective Hamiltonian: these are (we use Bjorken and Drell notations, definitions and conventions [6])

$$(u_L, d_L) \oplus (u_R, d_R) \oplus (s_L, s_R) \quad (3)$$

and their antiparticles. We assume input mass matrices:

$$M_{0n} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \quad (4)$$

Thus the simplest non-trivial relevant effective Hamiltonian can be assumed to be

$$\mathbf{H}_{st} = U_0 + \mathbf{H}_0 + \mathbf{V}^{(1)} + \mathbf{V}^{(2)} \quad (5)$$

$$\mathbf{H}_0 = \sum_{n=cu,cd,cs} \int d^3\vec{x} \bar{q}_{nR}(\vec{x}) (-i\vec{\gamma} \cdot \vec{\nabla}) q_{nR}(\vec{x}) + L \Longleftrightarrow R \quad (6)$$

$$\mathbf{V}^{(1)} = \sum_{n=cu,cd,cs} \int d^3\vec{x} \bar{q}_{nR}(\vec{x}) M_{0n} q_{nR}(\vec{x}) + L \Longleftrightarrow R \quad (7)$$

$$\mathbf{V}^{(2)} = \frac{g_{SP}}{8\pi\Lambda_\chi^2} \sum_{n,n'=cu,cd,cs} \int d^3\vec{x} \bar{q}_{nL}(\vec{x}) q_{nR}(\vec{x}) \bar{q}_{n'R}(\vec{x}) q_{n'L}(\vec{x}) + L \Longleftrightarrow R \quad (8)$$

$$+ \frac{g_{VA}}{8\pi\Lambda_\chi^2} \sum_{n,n'=cu,cd,cs} \int d^3\vec{x} \bar{q}_{nL}(\vec{x}) \gamma^\mu q_{nL}(\vec{x}) \bar{q}_{n'L}(\vec{x}) \gamma_\mu q_{n'L}(\vec{x}) + L \Longleftrightarrow R \quad (9)$$

The $q_{nL,R}$ are chiral field operators [6]. The necessary inputs for this model are:

- (i) a real flavour diagonal 3×3 "input mass matrix" M_{0n} ;
- (ii) Two real independent effective dimensionless couplings g_{SP} and g_{VA} ;
- (iii) two fundamental mass scales, provided by an explicit $\Lambda_\chi \sim 1\text{GeV}$ (related to the scale where massless quarks become massive quarks) and an implicit $\Lambda_{QCD} \sim 300\text{MeV}$ (a parameter basically fixing the overall size of physical hadrons). In the context of a non-chiral quark model this region would give the medium range $q\bar{q}$ potential.

III. THE RELEVANT LRO

Let us *define* the Landau-like long range order (LRO) parameter appropriate to an "u,d-quark condensate" by *assuming* the existence of a *robust* spinless, colourless and flavourless non-vanishing scalar LRO[2,3]

$$\Delta = \frac{g_{SP}}{8\pi\Lambda_\chi^2} \sum_{n=cu,cd,cs} < 0 | (\bar{q}_{nL}(\vec{x}) q_{nR}(\vec{x})) | 0 > + L \iff R \quad (10)$$

Translation invariance ensures independence on space coordinates. This *defines* g_{SP} . The notation $|0 >$ as used here should be explained. In a many-body context the symbol $|0 >$ would usually mean that one is referring to a certain state vector in Hilbert space representing the true ground state of the many-body system. In the context of this paper, such a true ground state of QCD-quark-gluon coupled fields is of course not only unknown but it is also irrelevant. This notation is nevertheless adopted here, but $|0 >$ merely defines a "no particle state" which is just a tautology for "normal operator products"[7]. It has therefore nothing to do with any true physical ground state. We shall sometimes refer to it rather loosely as the "quark vacuum".

We work exclusively with approximate Heisenberg operators that play the role of "physical states"[7]. It is established that only colour singlets would qualify as such. We try to find stationary solutions to the Heisenberg equations of motion for these "physical states". Thus our "no particle state" is nothing but a tautology for normal ordering DHFB quasiparticle operators.

In order to include this assumption in the effective Hamiltonian we begin by making a straightforward Bogoliubov transformation using the Nambu-Gorkov representation for the u,d QCD-quarks[5]:

$$\begin{pmatrix} \alpha_{n\lambda}(\vec{p}) \\ \beta_{\bar{n}\bar{\lambda}}^+(-\vec{p}) \end{pmatrix} = \sum_{h=\pm\frac{1}{2}} \begin{pmatrix} \sin \varphi_{nh\lambda}(p) & \cos \varphi_{nh\lambda}(p) \\ -\cos \varphi_{nh\lambda}(p) & \sin \varphi_{nh\lambda}(p) \end{pmatrix} \begin{pmatrix} b_{nh}(\vec{p}) \\ d_{\bar{n}h}^+(-\vec{p}) \end{pmatrix} \quad (11)$$

where $\lambda = L, R$ are chiralities and h are helicities. The "Bogoliubov angles $\varphi_{nh\lambda}(p)$ " serve as adjustable variational parameters. By separating out the bilinear from the non-bilinear terms we find that

$$\mathbf{H}_{st}(u, d, s) = U'_0 + \mathbf{H}'_0 + \mathbf{V}' \quad (12)$$

where

$$\mathbf{H}'_0 = \sum_{n=cu,cd,cs} \int d^3\vec{p} : \begin{pmatrix} \bar{q}_{nR}(\vec{p}) & \bar{q}_{nL}(\vec{p}) \end{pmatrix} \begin{pmatrix} |\vec{p}| & -\Delta_n \\ -\Delta_n & -|\vec{p}| \end{pmatrix} \begin{pmatrix} q_{nL}(\vec{p}) \\ q_{nR}(\vec{p}) \end{pmatrix} \quad (13)$$

and \mathbf{V}' represents the remaining (quadrilinear) terms. The Bogoliubov angles are chosen so that \mathbf{H}'_0 is fully diagonalized:

$$\mathbf{H}'_0 = -U'_0 + \sum_{n=cu,cd,cs} \sum_{h=\pm\frac{1}{2}} \int d^3\vec{p} E_n(p) [b_{nh}^+(\vec{p}) b_{nh}(\vec{p}) + d_{\bar{n}h}^+(-\vec{p}) d_{\bar{n}h}(-\vec{p})] \quad (14)$$

$$\sin^2 \varphi(p) = \frac{1}{2}(1 - |\vec{p}|/E_{u,d}(p)) \quad n = u, d \quad (15)$$

$$E_{u,d}(p) = \sqrt{|\vec{p}|^2 + (m_{u,d} + \Delta)^2} \quad (16)$$

$$E_s(p) = \sqrt{|\vec{p}|^2 + m_s^2} \quad (17)$$

where by definition

$$b_{nh}(\vec{p})|0\rangle = 0 = d_{\bar{n}h}(-\vec{p})|0\rangle \quad (18)$$

Note the independence of the Bogoliubov angles on helicities /chiralities that follow from the definition of the LRO. We shall refer to this as the Dirac-Hartree-Fock-Bogoliubov(DHFB) static approximation.

IV. RENORMALIZATIONS

We shall have to renormalize the above theory, selecting experimental data on pions and the etas for doing so[1]. These mesons, just like any physical mesons, are here considered to be just complex poles of the physical S-matrix amplitudes.

The electromagnetic sector of this model is represented by the Hamiltonian

$$\mathbf{H}_{ew} = \mathbf{H}_{0l\gamma} + \mathbf{V}_{ew} \quad (19)$$

and will be treated perturbatively. $\mathbf{H}_{0l\gamma}$ stands for free leptons and gammas. The \mathbf{V}_{ew} is given by the SM[2,3].

(i) Consider the π^\pm main decay channel in the rest frame, defining the decay constant f_{π^\pm}

$$A(\pi^\pm \longrightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)) \equiv \langle 0 | J_{weak} | \pi^\pm(M_{\pi^\pm}) \rangle > \quad (20)$$

where

$$|\pi^\pm(M_{\pi^\pm}) \rangle = |\pi^\pm \rangle \equiv \frac{1}{\sqrt{6}} \sum_c \sum_h \int d^3\vec{p} \Psi_{\pi^\pm}(\vec{p}) * b_{cuh}^+(\vec{p}) d_{cdh}^+(-\vec{p}) |0 \rangle \quad (21)$$

$$|\langle 0 | J_{weak} | \pi^\pm(M_{\pi^\pm}) \rangle| = |f_{\pi^\pm}| \sqrt{\frac{M_{\pi^\pm}}{2(2\pi)^3}} \quad (22)$$

we find the condition

$$\sqrt{\frac{M_{\pi^\pm}}{3(2\pi)^3}} \frac{|f_{\pi^\pm}|}{4 \cos \theta_C} = \left| \int \frac{d^3\vec{p}}{(2\pi)^3} \Psi_{\pi^\pm}(\vec{p}) * \cos 2\varphi(p) \right| \quad (23)$$

(ii) Consider the π^\pm charge radius, defined through

$$\langle \pi^+(\vec{p}_2) | J_{em}^\mu(0) | \pi^+(\vec{p}_1) \rangle = G_\pi((p_1 - p_2)^2) \frac{(p_1 + p_2)^\mu}{\sqrt{(2\pi)^3 2E_\pi(p_2)(2\pi)^3 2E_\pi(p_1)}} \quad (24)$$

where

$$G_\pi((p_1 - p_2)^2) = 1 + \frac{1}{6} \langle r_\pi^2 \rangle (p_1 - p_2)^2 \quad (25)$$

A simple estimate of the theoretical charge radius can be obtained by making a reasonable ansatz for the (common) normalized internal wavefunction of π^\pm :

$$\Psi_{\pi^\pm}(\vec{p}; x, y) = \frac{1}{\sqrt{4\pi}} N(x, y) \exp\left(-\frac{1}{2}(\rho - x\Lambda_\chi^{-1})^2 / (y\Lambda_{QCD}^{-1})^2\right) \quad (26)$$

where x, y are dimensionless variational parameters. So

$$\langle r_\pi^2 \rangle = \Lambda_\chi^2 \frac{F_4(x, y; \Lambda_\chi / \Lambda_{QCD})}{F_2(x, y; \Lambda_\chi / \Lambda_{QCD})} \quad (27)$$

$$F_j(x, y; \Lambda_\chi/\Lambda_{QCD}) = \int_{-x/y}^{\infty} du \exp(-(\Lambda_\chi/\Lambda_{QCD})^2 u^2) (x + yu)^j \quad (28)$$

(iii) Next, consider the main decay channel of the π^0 at rest:

$$A(\pi^0 \longrightarrow \gamma(\vec{k}) + \gamma(-\vec{k})) = \langle \gamma(\vec{k}), \gamma(-\vec{k}) | \mathbf{V}_{em} | \pi^0 \rangle \quad (29)$$

$$|\pi^0 \rangle = \frac{1}{\sqrt{12}} \sum_c \sum_{q=cu, cd} \sum_h \int d^3\vec{p} \Psi_{\pi^0}(\vec{p}) b_{cq}^+(\vec{p}) d_{c\bar{q}h}^+(-\vec{p}) |0 \rangle \quad (30)$$

But assuming the initial pseudoscalar at rest, we find that the absorptive part of the (dominant) underlying Lorentz and gauge invariant $q\bar{q}$ annihilation amplitude a (with massive quarks) is :

$$k^2 \text{Im } a(\pi^0(\longleftrightarrow q + \bar{q}) \longrightarrow \gamma(\vec{k}) + \gamma(-\vec{k})) \sim \sim (\Delta/k)^2 (1 - (\Delta/k)^2)^{-\frac{1}{2}} \ln[1 - (1 - (\Delta/k)^2)^{\frac{1}{2}} / (1 + (1 - (\Delta/k)^2)^{\frac{1}{2}})] \quad (31)$$

As $\Delta, k \longrightarrow 0$ it can be shown that the lhs of (31) tends to $\delta(k^2)$ [8], contrary to naïve expectations. The deep reason for this is the existence of the fixed anomaly pole at $k = 0$, a necessary feature if the U_{1em} gauge invariance is to be maintained. This pole lives below the physical threshold (at about $k = 2\Delta$), which can be related to the physical mass of the π^0 through the Gell-Mann-Oakes-Renner formula [2,3]. The existence of this pole could be guessed from (31) as the rise of $k^2 \text{Im } a$ as k descends from infinity towards the physical threshold. This provides another condition on our parameters.

(iv) In order to calculate the value of the $\eta_0 - \eta'_0$ anomaly [2,3] we shall have to include further interaction terms from (12). We shall consider the effect only to leading order and ignore all isospin mixings. Let us define

$$|\eta_i \rangle = \frac{1}{\sqrt{3}} \sum_c \sum_h \sum_{f_i} \int d^3\vec{p} \Psi_{\eta_i}(\vec{p}) b_{cf_i h}^+(\vec{p}) d_{c\bar{f}_i h}^+(-\vec{p}) |0 \rangle \quad i = 1, 2 \quad (32)$$

with $f_1 = u, d$ and $f_2 = s$.

So by diagonalizing the 2×2 matrix

$$M_{ij} = \langle \eta_i | \mathbf{H}_{st}(u, d, s) - U_0 | \eta_j \rangle \quad (33)$$

we find that the eigenvectors (in the approximation of keeping only the $q\bar{q}$ pair exchange diagrams) are

$$|\eta_0 \rangle = \cos \theta |\eta_1 \rangle + \sin \theta |\eta_2 \rangle \quad (34)$$

$$|\eta'_0 \rangle = \sin \theta |\eta_1 \rangle - \cos \theta |\eta_2 \rangle \quad (35)$$

where the mixing angle is given by

$$\tan \theta = (M_{\eta^0} - M_{11})/M_{12} = M_{21}/(M_{\eta'_0} - M_{22}) \quad (36)$$

$$M_{11} = 2E_{u,d} + |F_1|^2 \quad M_{22} = 2E_s + |F_2|^2 \quad M_{12} = F_1^* F_2 + F_1 F_2^* = M_{21} \quad (37)$$

$$F_1 \equiv \frac{1}{\sqrt{12}} \int_0^\infty d^3 \vec{p} \Psi_{\eta_1}(\vec{p}) A_1(\Delta, \vec{p}) \quad F_2 \equiv \frac{1}{\sqrt{6}} \int_0^\infty d^3 \vec{p} \Psi_{\eta_2}(\vec{p}) A_2(M_s, \vec{p}) \quad (38)$$

$$A_1(\Delta, \vec{p}) \equiv 12\Delta / \sqrt{|\vec{p}^2| + \Delta^2} \quad (39)$$

$$A_2(M_s, \vec{p}) = 6M_s / \sqrt{|\vec{p}^2| + M_s^2} \quad (40)$$

The angle θ is thus part of our renormalized parameters.

V. CONCLUSIONS

We presented an Hamiltonian field theory in a DHFB-static approximation as a complement/alternative to the conventional theory of pions and etas. However, our theory (based on an updated version of the NJL field theory[9]) can be easily extended and has a much broader scope. It can also easily be linked to non-chiral quark models and QCD in the deep infrared. All renormalizations procedures are build in the theory itself and have of course only meaning in the context of this theory, as every renormalization scheme does in

its own context. The issue of confinement, though not essential to our case, is bypassed in a natural way. Detailed numerical fits to experimental data in order to get the renormalized parameters will be published elsewhere.

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